# Grade 7/8 Math Circles

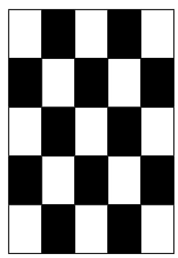
## February 5 & 6 & 7 & 8, 2024

## The Pigeonhole Principle - Problem Set

1. I have 4 green shirts, 2 black shirts, and 3 blue shirts in my closet. Suppose that I take out shirts without looking at them. How many shirts do I have to take out to be sure that there are two shirts of the same colour?

2. In molecular biology, organisms have a genetic code made up of 61 distinct codons (a sequence of three *nucleotides*) that each code for an amino acid. Each of these codons binds to a tRNA molecule. Most organisms have fewer than 45 types of tRNA. How is this possible?

3. A group of fleas are playing a version of musical chairs on a $5 \times 5$ board.



   (a) Suppose that there is a flea on every black square of the $5 \times 5$ board above, and no fleas on the white squares. Is it possible for the fleas to all hop to a white square such that no two fleas are on the same white square? Justify your answer.

   (b) Suppose that there is a flea on every white square of the $5 \times 5$ board above, and no fleas on the black squares. Is it possible for the fleas to all hop to a black square such that no two fleas are on the same black square? Justify your answer.

4. 5 people on Earth participate in a Zoom call. None of them are on the equator. Show that there are at least 3 people who are in the same (north or south) hemisphere.

5. Jeff talked about one of three topics on each of 16 days. Show that there are at least 6 days on which Jeff talked about the same topic.

6. There are 50 baskets of apples. Each basket contains no more than 23 apples. Show that there are 3 baskets with the same number of apples. *Hint: What are the categories in this scenario?*

7. There are 3367 people at a music concert. Each person has skydived between 0 to 34 times. What's the maximum number of people that we can guarantee has skydived the same number of times?

8. In an arbitrary list of 89 odd numbers, how many can we guarantee end in the same last digit?

9. Tanya has a list of 2025 different prime numbers. Brent says that there cannot be two numbers on Tanya's list that differ by a multiple of 2024. Margaret says that there has to be two numbers on Tanya's list that differ by a multiple of 2024. Who is correct? Use the Pigeonhole Principle to justify your answer. *Hint: If the remainders after diving two numbers $a$ and $b$ by $c$ are the same, then what can we say about $a$ and $b$?*

10. Suppose that George picks 5 different numbers from the integers 1 to 8. Use the Pigeonhole Principle to show that two of the numbers must add up to 9.

    (a) Determine a set of categories to place each of the five numbers into when applying the Pigeonhole Principle.

    (b) Using the Pigeonhole Principle, show that one of the categories that you determined in (a) contains at least two of the five numbers.

11. If 18 people are seated in a row of 21 chairs, then what is the maximum number of consecutive chairs that are guaranteed to be occupied?

12. Suppose that Bailey picks 10 different numbers out of the integers 1, 2, . . . , 116. He writes each of the numbers that he picks onto a separate marble and places these marbles in a bag. Show that one can draw the marbles in two different ways such that the sum of the marbles in either draw is the same. ("Drawing the marbles" means pulling out a selection of 0 to 10 marbles from Bailey's bag.)

    (a) How many different ways are there to draw marbles from the bag? *Hint: This is the number of subsets of a set with 10 elements. Read more about subsets of a set here: mathisfun.com/activity/subsets.html.*

    (b) Show that there are more ways to draw marbles from the bag than the number of possible sums.

    (c) Why does showing that (b) is true prove that it is possible to draw the marbles in two different ways such that the sum of the marbles in either draw is the same?

    *Note that this problem can be extended to show that there is a way to pick some marbles into one group and pick some marbles into another group (with the two groups having no overlapping marbles) such that the two groups of marbles have the same sum. See the 1972 International Mathematical Olympiad, question 1.*